A bioeconomic MPA study based on

Cellular automata population growth and distribution

A theoretical study

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Ouline

- **Introduction**
- **Biological models**
- **Economic model**
- **Results**
- **Conclusion**

Basic assumptions

- **MPA in this presentation is identical to marine sanctuary: A defined region where fishing activities are prohibited**
- **Perfect control, no control costs**
- **Open access to non-protected areas**
- **Biological dynamics determined by fish behaviour and net growth at micro level (bottom - up)**

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Bioeconomic MPA studies

- **The value of MPA as a management measure is disputed**
- **Deterministic surplus production models indicate that MPAs have limited impact on stock development and fishing effort, unless the MPAs are very large**
- **The main challenge seems to be to model the spatial component in a useful way**

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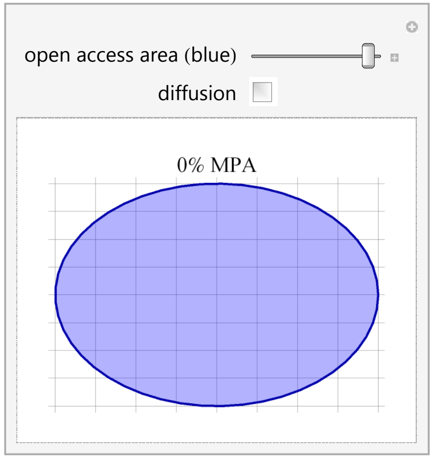
Standard approach for modelling MPAs



– Standard approach is to modify aggregated surplus production models, separating the stock biomass into two area specific groups

– The overall migration is usually assumedto be driven by the biomass gradient between the two areas (sectors)

Section 1

The biological model

Two alternative biological models:

- **Continous totalistic CA**
- **Logistic growth (discrete time version)**

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Basics

Problems to solve

- **Density driven migration has to be defined locally, while standard MPA models assume fish to react on density gradients at macro level**
- **As long as a density gradient exists within the open area, spatial distribution of fishing activities does matter**

Rational behaviour (of fish and fisher)

- **Growth and replacement in space are essentially micro level processes:**
- **Individual growth and mortality are functions of locally available food and presence of predators**
- **Each fish individual moves in the direction of the food source and flee from the predators**
- **High density of fish may cause less food per individual and at the same time attract predators**
- **If possible, the fisher would approach non-protected areas with the highest fish density**

Biomass distribution

Assume a linear representation of a fish stock biomass distribution. The initial biomass vector with n elements is

$\mathbb{b} = (b_1, b_2, b_3, \ldots., b_n).$

Capacity limits

$\mathbb{b}$ evolves over time as a function of $\mathbb{b}$ and a simple CA rule involving a growth rate (g) and a fixed diffusion pattern.
The diffusion pattern is given by the range parameter r.

$0 \leq b_i \leq 1$ for $1 \leq i \leq n$.

The growth rate (g) gives the percentage growth per unit of time.

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Fractional growth model

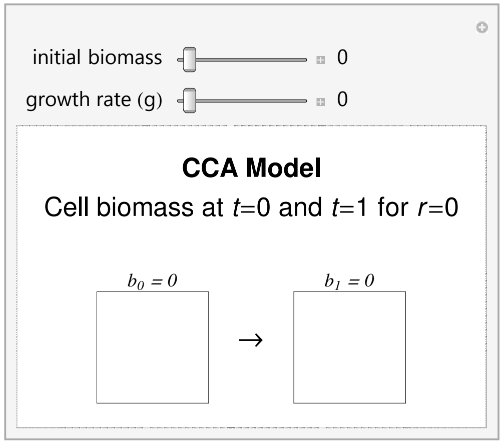
The biomass growth is expressed by

$$b_{i,\,t+1} = \operatorname{frac}\left(\frac{g+1}{2\,r+1}\sum_{j=i-r}^{i+r} b_{j,t}\right),$$

while $b_{n+1,\,t} = b_{1,t}$, $g \geq 0$ and $r \geq 0$.

Natural mortality is expressed indirectly by the remaining fractional part, assuming a density dependent mortality.
The biomass vector is a discrete function of time at given initial biomass value ($\mathbb{b}_0$), here expressed by the continuous cellular automata rule

$$\mathbb{b}_t = \mathrm{CCA}(\mathbb{b}_{t-1}).$$



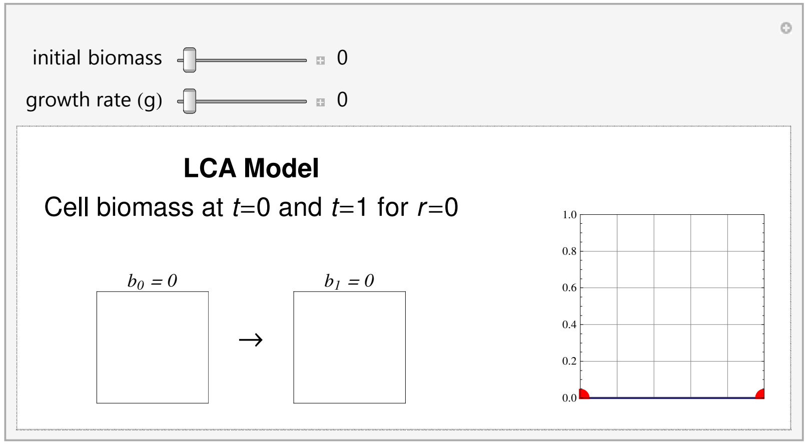
Logistic growth model

Corresponding discrete time logistic growth equation (difference equation) is

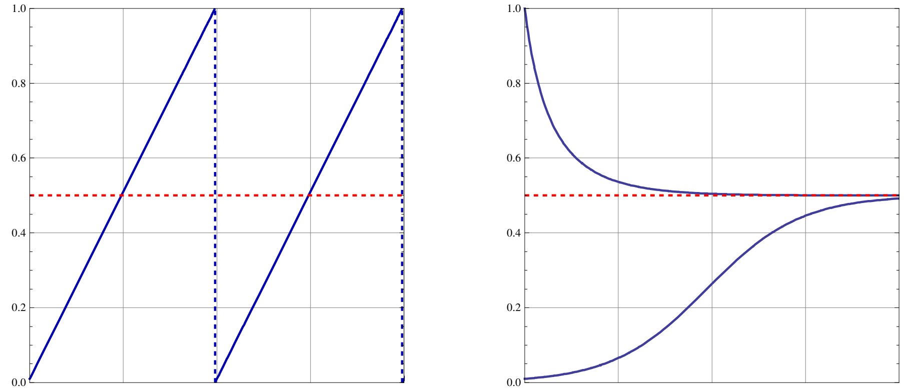
$$b_{i,\,t+1} = \frac{g+1}{2\,r+1}\left(1 - 2\sum_{j=i-r}^{i+r} b_{j,t}\right)\sum_{j=i-r}^{i+r} b_{j,t},$$

referred to as cellular automata rule

$$\mathbb{b}_t = \mathrm{LCA}(\mathbb{b}_{t-1}).$$



CCA vs. LCA



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Total biomass

Total biomass at time t is

$$B_t = \sum_{i=1}^{n} b_{i,\,t}.$$

Equilibrium biomass is $B_\infty = \frac{n}{2}$ for $r > 0$.

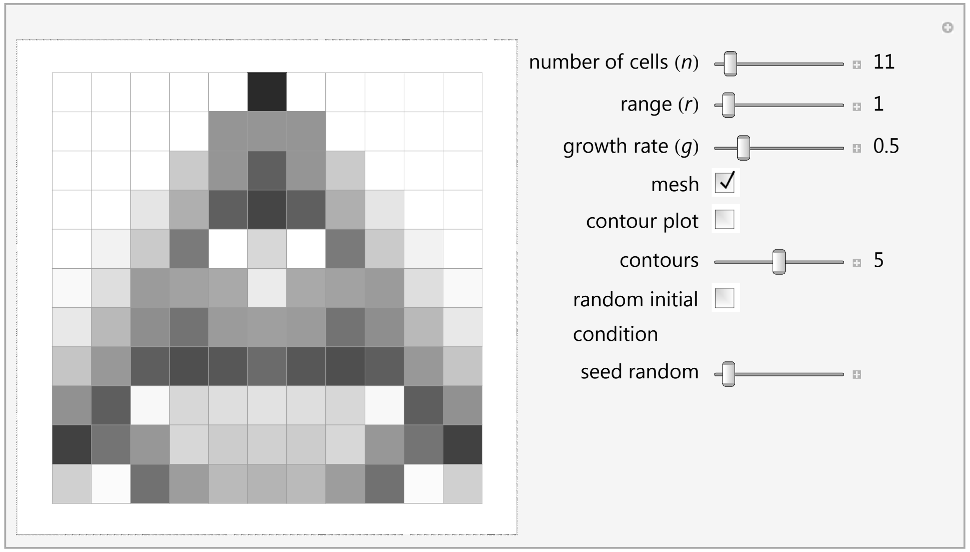
CCA results of n=13, r=1, g=$\frac{1}{2}$, initial value 1 placed in cell 7

	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}	b_{13}	B
t=0	0	0	0	0	0	0	1	0	0	0	0	0	0	1
t=1	0	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	0	$\frac{3}{2}$
t=2	0	0	0	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0	0	0	0	$\frac{9}{4}$
t=3	0	0	0	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{1}{8}$	0	0	0	$\frac{27}{8}$
t=4	0	0	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{5}{8}$	0	$\frac{3}{16}$	0	$\frac{5}{8}$	$\frac{1}{4}$	$\frac{1}{16}$	0	0	$\frac{33}{16}$
t=5	0	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{15}{32}$	$\frac{7}{16}$	$\frac{13}{32}$	$\frac{3}{32}$	$\frac{13}{32}$	$\frac{7}{16}$	$\frac{15}{32}$	$\frac{5}{32}$	$\frac{1}{32}$	0	$\frac{99}{32}$
t=6	$\frac{1}{64}$	$\frac{3}{32}$	$\frac{21}{64}$	$\frac{17}{32}$	$\frac{21}{32}$	$\frac{15}{32}$	$\frac{29}{64}$	$\frac{15}{32}$	$\frac{21}{32}$	$\frac{17}{32}$	$\frac{21}{64}$	$\frac{3}{32}$	$\frac{1}{64}$	$\frac{297}{64}$

CCA results of n=7, r=1, g=$\frac{3}{5}$, initial value 1 placed in cell 4

	b_1	b_2	b_3	b_4	b_5	b_6	b_7	B
t=0	0	0	0	1	0	0	0	1
t=1	0	0	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$	0	0	$\frac{6}{5}$
t=2	0	$\frac{24}{125}$	$\frac{128}{375}$	$\frac{56}{125}$	$\frac{128}{375}$	$\frac{24}{125}$	0	$\frac{568}{375}$
t=3	$\frac{7616}{78125}$	$\frac{832}{3375}$	833 1⋱ 52 / 2 1⋱ 09 ⋱ 375	912 4⋱ 48 / 2 1⋱ 09 ⋱ 375	833 1⋱ 52 / 2 1⋱ 09 ⋱ 375	$\frac{832}{3375}$	$\frac{7616}{78125}$	4 030 ⋱ 016 / 2 1⋱ 09 ⋱ 375

Basic CCA model



Outputs from one initial cell of biomass 0.5

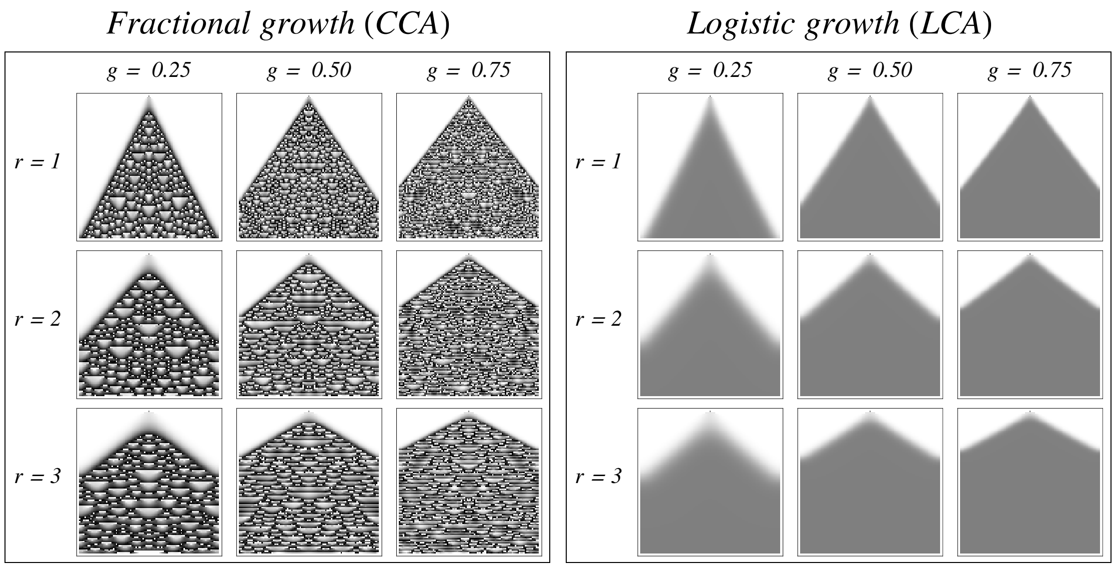


Figure 1. CCA model (4) of varying diffusion properties at constant growth with an initial condition of one single biomass (b_{49}=1) in the centre cell of 99 cells (n = 99). The growth rate (g) is 0.5 and the diffusion property given by the range parameter r , indicating number of influenced neighbouring cells. The vertical axes of each case represents computational steps (time), increasing downwards. The figure includes 100 computational steps (t=100).

Outputs from initial random biomass distribution

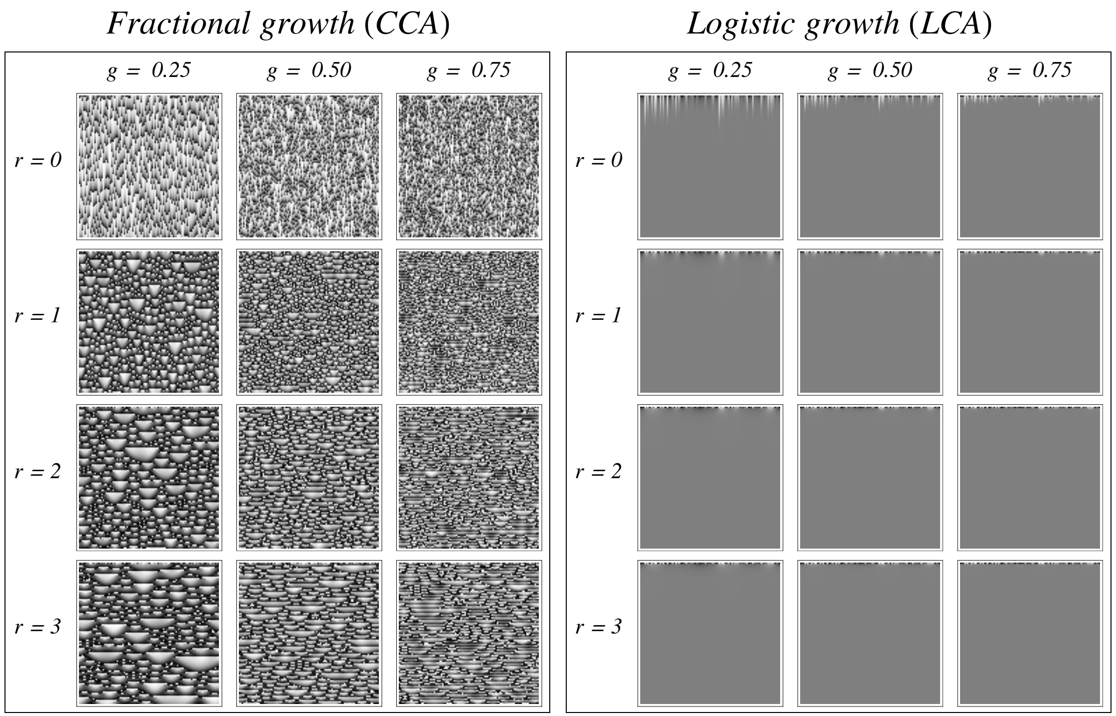


Figure 2. CCA model (4) of varying diffusion properties (*r*) at growth rates (*g*) as explained in Figure 1. Each panel has the same seed random initial cell biomasses distributed in 100 cells ($n = 100$). The vertical axes of each case represents computational steps (time), increasing downwards. The figure includes 100 computational steps (t=100).

Corresponding total biomass (1)

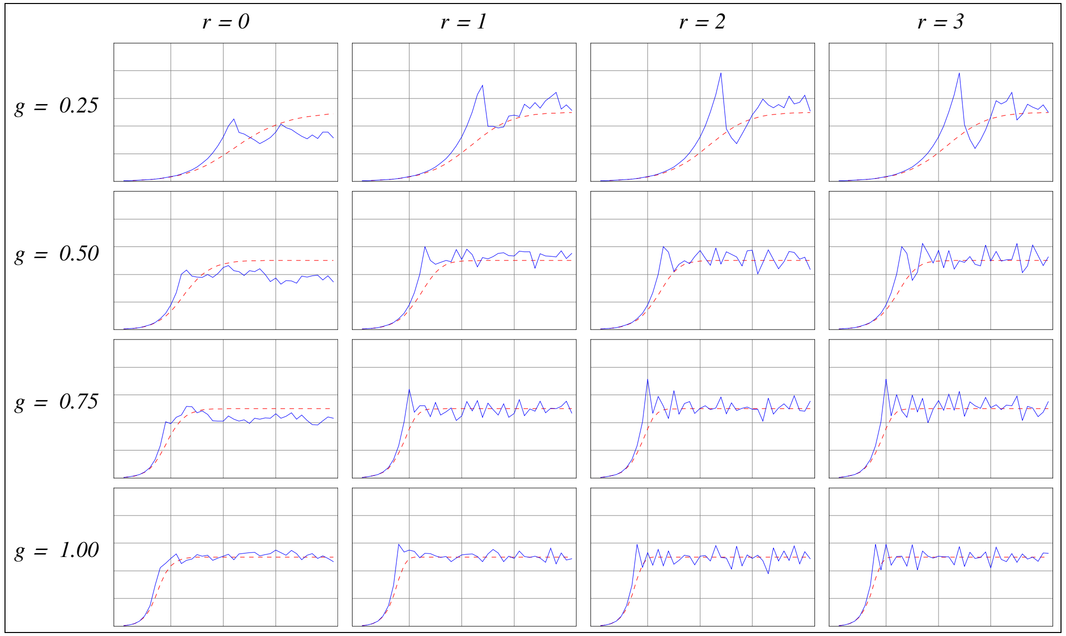


Figure 4. Biomasses over time in a CCA model (4) (solid curves) and LCA model (7) (dashed curves) of varying diffusion properties (*r*) at growth rates (*g*) with the same random initial cell biomasses and 100 cells (n = 100). The vertical axes of each case measures biomass and the horizontal axis time. The figure includes 100 time steps (t=100).

Corresponding total biomass (2)

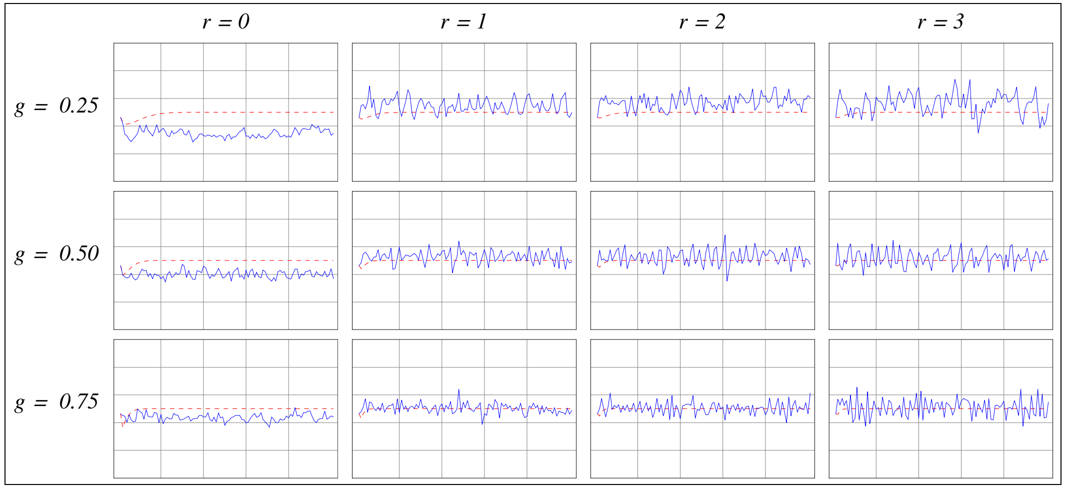
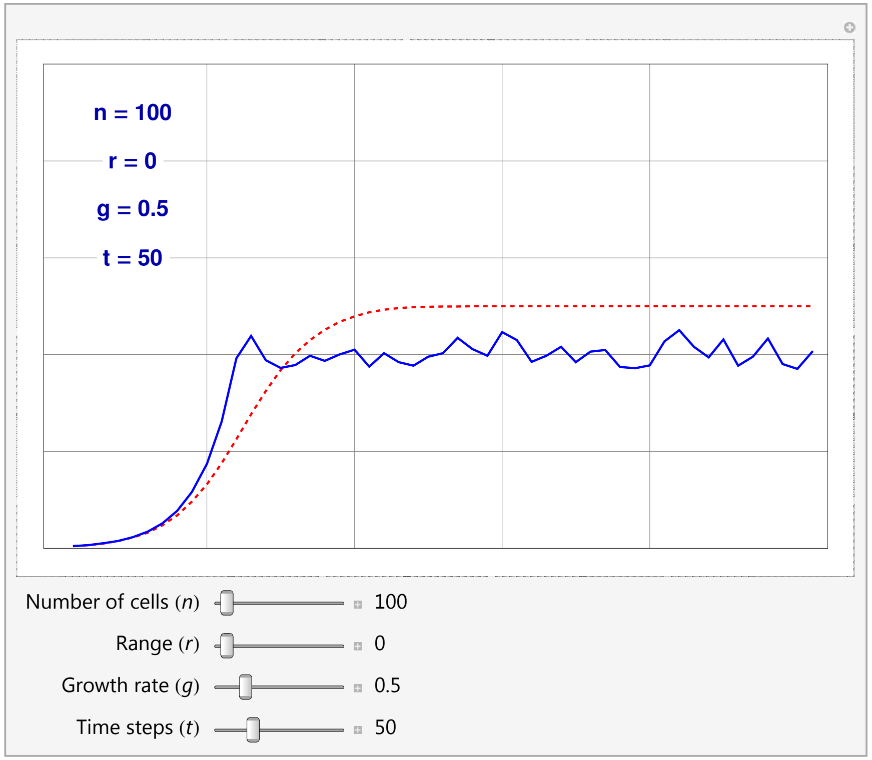


Figure 3. The solid curves represent biomasses over time derived by CCA model (4) of range parameter values (*r*) and growth rates (*g*) with the same seed random initial cell biomasses distributed in 100 cells ($n = 100$). The vertical axes of each case measures biomass and the horizontal axis time. The figure includes 100 computational steps (t=100). The dashed curves are biomasses derived by LCA model (7), the corresponding logistic growth functions.

Parameters and variables of the two growth models (global picture)



Section 2

The economic model

Two alternative biological models:

- **Fisheries management (Marine Protected Area)**
- **Harvest equation**
- **Revenue and cost of fishing**

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Fishing regulated by closed area

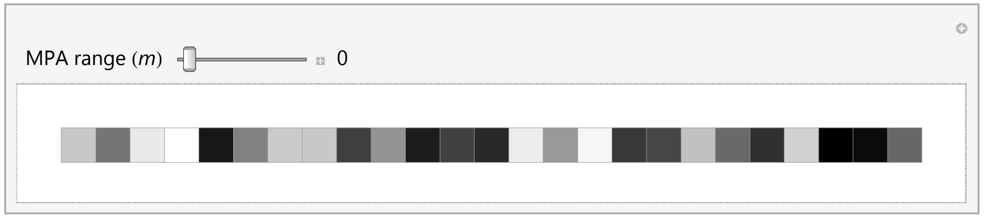
The stock biomass within a *MPA* is given as a subset of biomass vector $\mathbb{b}$,

$$\mathbb{b}_{\text{MPA}} = (b_s, \; ...., \; b_{s+m-1})$$

where s is the first cell and m is the number of cells included in the *MPA*. Absence of protected area is regarded being a special case of *MPA* regulation (no closed area; $m = 0$). The model circularity makes the choice of s-value unimportant, hence $s = 1$ is used in the following. The *MPA* biomass vector then simplifies to

$$\mathbb{b}_{\text{MPA}} = (b_1, \; ...., \; b_m),$$

$$0 \leq m \leq n$$



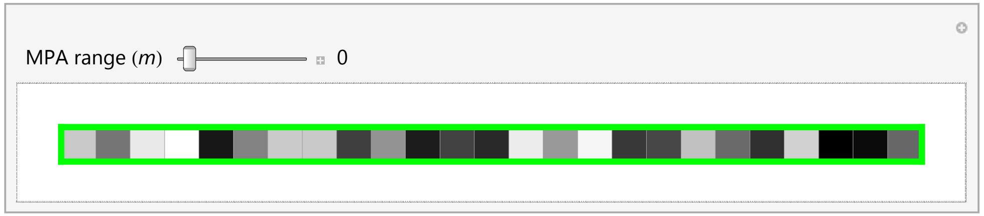
Biomass exposed to harvest

Fishing activities target biomasses in the non-protected area (*NPA*), represented by the complementary subset to $\mathbb{b}_{\text{MPA}}$ of the biomass vector $\mathbb{b}$

$$\mathbb{b}_{\text{NPA}} = (b_{1+m}, \; ...., \; b_n).$$

Total targeted biomass at time t then is

$$B_{\text{NPA},t} = \sum_{i=1+m}^{n} b_{i,\,t}.$$



Harvest model

The fish harvest production function used in this study assumes a stock output elasticity of 1/2, while harvest (h) is assumed to be linear in fishing effort;

$$h_{i,t} = q\, e_{i,t} \sqrt{b_{i,t}}\,,$$

when $b_i \in \mathbb{b}_{\mathrm{NPA}}$ and e_i is the fishing effort of cell i. Total fishing effort is the sum of the fishing effort of all cells

$$E_t = \sum_{i=1+m}^{n} e_{i,\,t}$$

total harvest is given by

$$H_t = \sum_{i=1+m}^{n} h_{i,\,t}\,.$$

Effort distribution

Rule of effort distribution based on distribution of stock biomass:

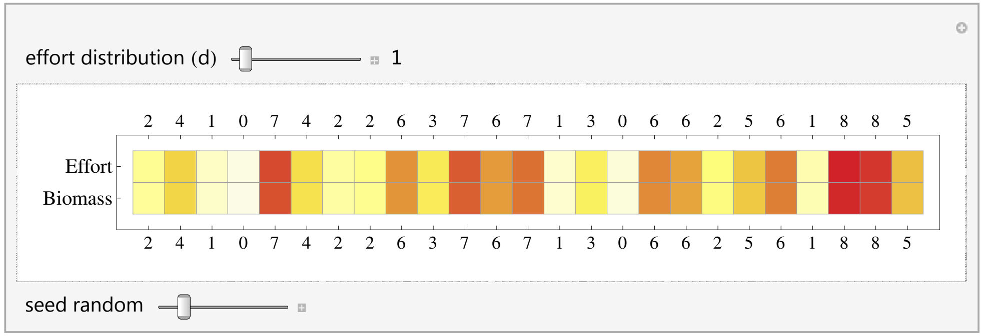
$$e_{i,t} = \frac{{b_{i,t}}^d}{\sum_{i=1+m}^{n} {b_{i,t}}^d} E_t,$$

where d is the distribution parameter (normally $d \geq 0$ is expected).

$d = 0$ gives a homogenous distribution of fishing effort:

$$e_{i,t} = \frac{{b_{i,t}}^0}{\sum_{i=1+m}^{n} {b_{i,t}}^0} E_t = \frac{E_t}{n-m},$$

while $d = 1$ mirrors the distribution of effort (below $m = 0$):



CA model including harvest

Including the harvest model in the biological growth of *CCA* yields

$$b_{i,\,t+1} = \text{frac}\left(\frac{g+1}{2\,r+1}\sum_{j=i-r}^{i+r} b_{j,t}\right) - h_{i,t}$$

The complete *CCA* model including harvest (by the fishing effort *E*) and *MPA* regulation (by the *MPA* size variable *m*), is expressed by

$$\mathbb{b}_t = \text{CCA}(\mathbb{b}_{t-1},\, E_{t-1},\, m_{t-1},),$$

m being the number of *MPA* cells and *E* the total fishing effort. Corresponding expression in the case of logistic growth is

$$b_{i,\,t+1} = \frac{g+1}{2\,r+1}\left(1 - 2\sum_{j=i-r}^{i+r} b_{j,t}\right)\sum_{j=i-r}^{i+r} b_{j,t} - h_{i,t}$$

and the *LCA* rule is modified accordingly to

$$\mathbb{b}_t = \text{LCA}(\,\mathbb{b}_{t-1},\, E_{t-1},\, m_{t-1},).$$

Economic model

The harvest equation (11) involves fishing effort (E) which is assumed to have a fixed unit cost c. The unit cost c also includes opportunity costs of all input factors in the production of fishing effort. Similarly a constant unit price of harvest (p) is assumed. The net revenue of harvest (NR) is

$$NR = p\,H - c\,E.$$

Since a normal profit is included in the unit cost of effort, NR more precisely is the total resource rent obtained in the fishery.

Economic Rent

Economic rent is earned when $NR > 0$, which in open access is expected to lead to an increase in fishing effort. The increase is assumed to be linear in rent:

$$E_{t+1} = E_t\,(1 + a\,NR_t).$$

a is the adjustment (stiffness) parameter and represents the intrinsic rate of change in effort. Cost of fishing effort (c) is decomposed on cells by fishing effort, and net revenue (economic rent) of each cell i is

$$nr_i = p\,h_i - \frac{c\,b_i^d}{\sum_{i=1+m}^{n} b_i^d}\,E$$

or simply

$$nr_i = p\,h_i - \frac{c\,E}{n-m}$$

in case of $d = 0$. Global net revenue is

$$NR = \sum_{i=1}^{n} nr_i.$$

Section 3

Results

The bioeconomic cellular automata model includes

- **biological and spatial parameters:** *g*, *r*, *d* **and** *n*
- **economic parameters:** *q*, *p*, *c* **and** *a*; **while** *m* **is a managment variable**
- **two state variables: biomassvector** 𝕓 **and effort vector** E

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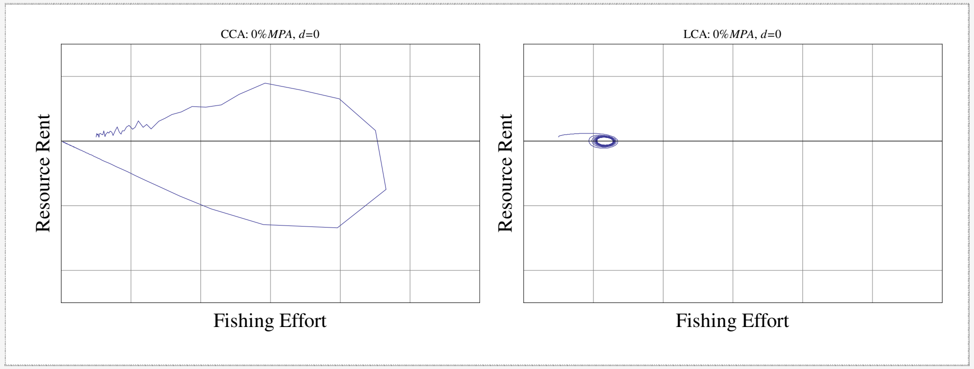
Resource Rent



n 100

t 500

m 0

g 0.5

r 2

d 0

e0 5

a 0.002

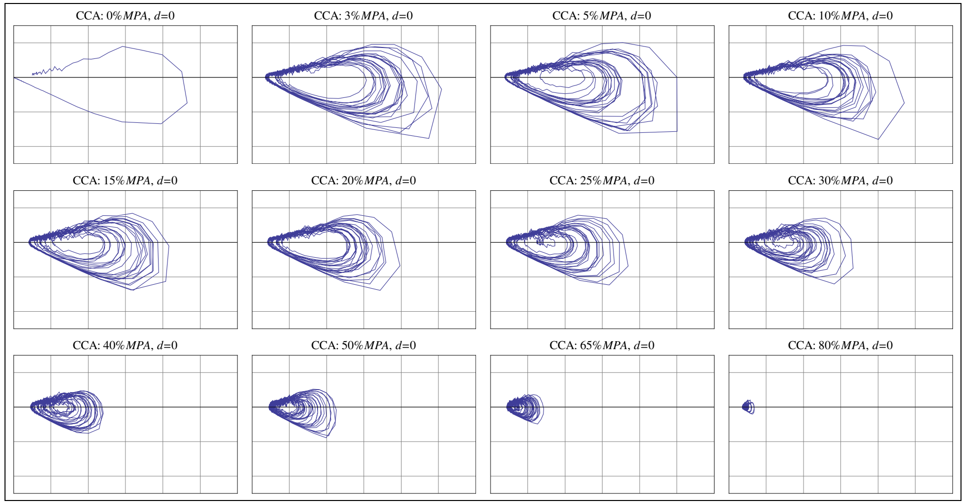
p 10

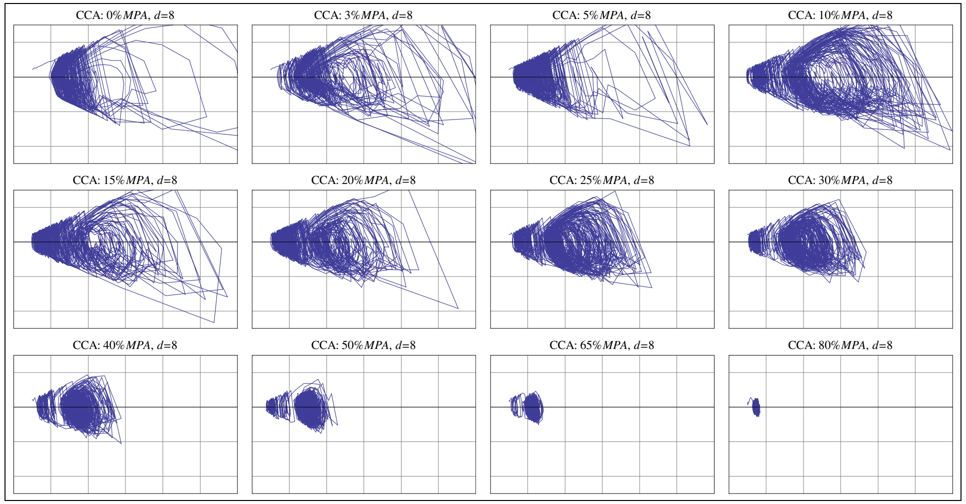
c 5

Chosen Parameter Values

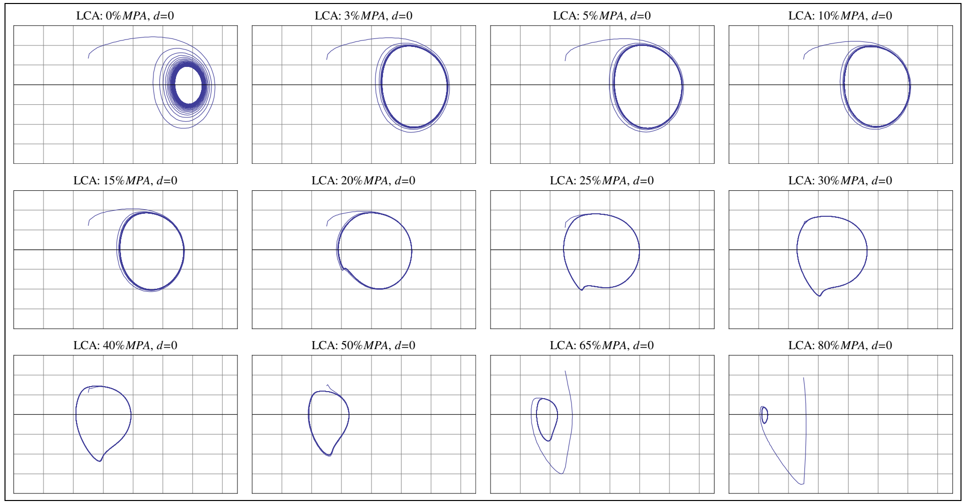
Parameter	CCA	LCA	Comment
t	2000	2000	Number of time steps
n	100	100	Number of cells in population
r	2	2	Number of affected neighbouring cells on each side
g	0.5	0.5	Biological growth rate
$\mathbb{b}_0$	Seed random	Seed random	Initial cell biomass vector
E_0	5	5	Initial fishing effort
d	0 and 8	0 and 8	Fishing effort distribution parameter
q	1	1	Catchability coeffisient
p	10	10	Unit price of harvest
c	5	5	Unit cost of effort
a	0.002	0.002	Growth rate in effort dynamics

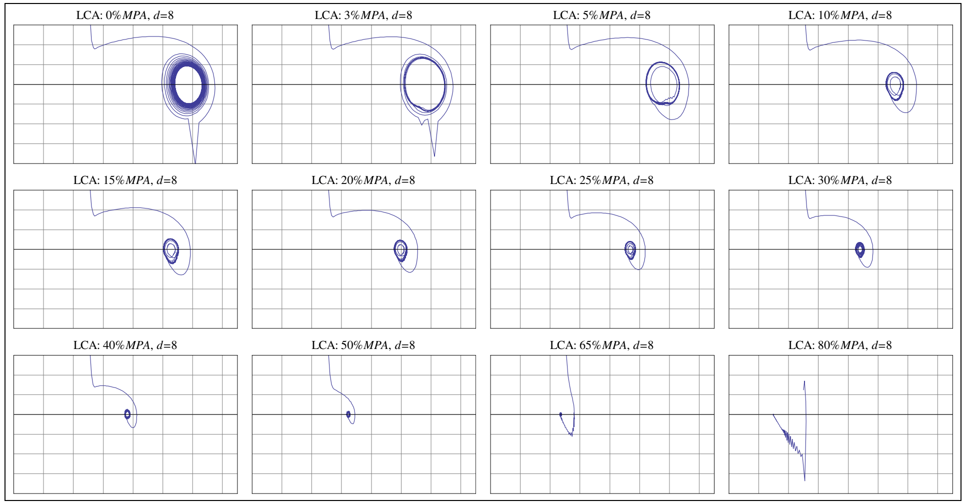
Quasi Rent in CCA



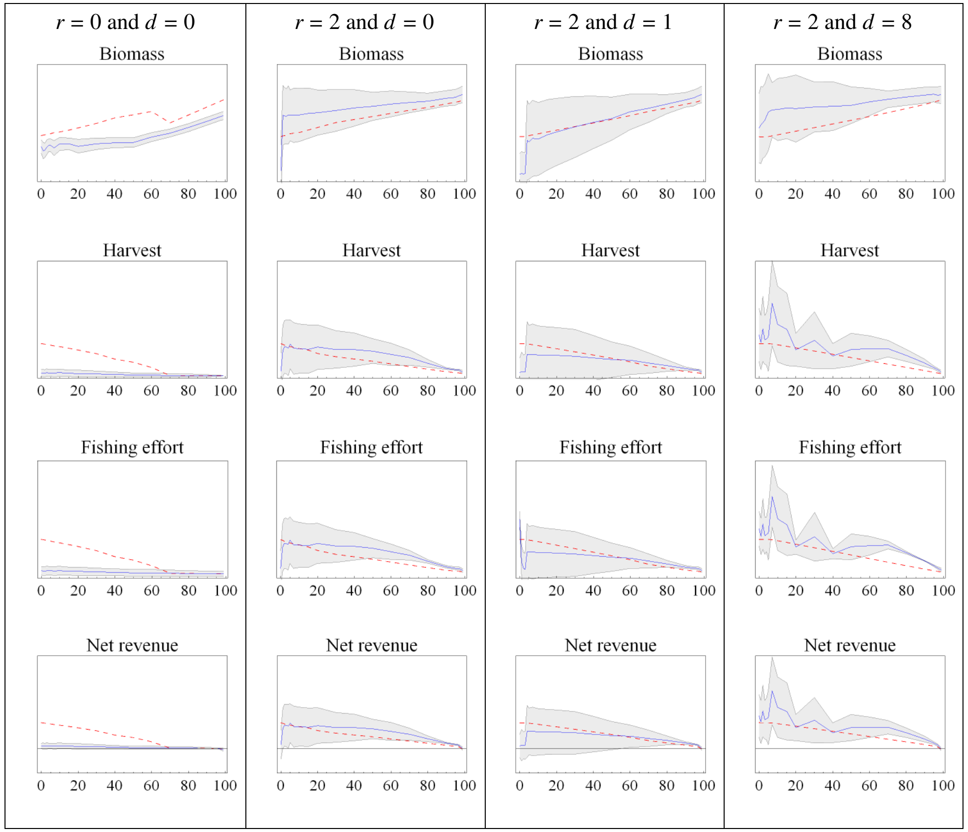


Quasi Rent in LCA





Effects of varying MPA size



CCA biomass

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LCA biomass

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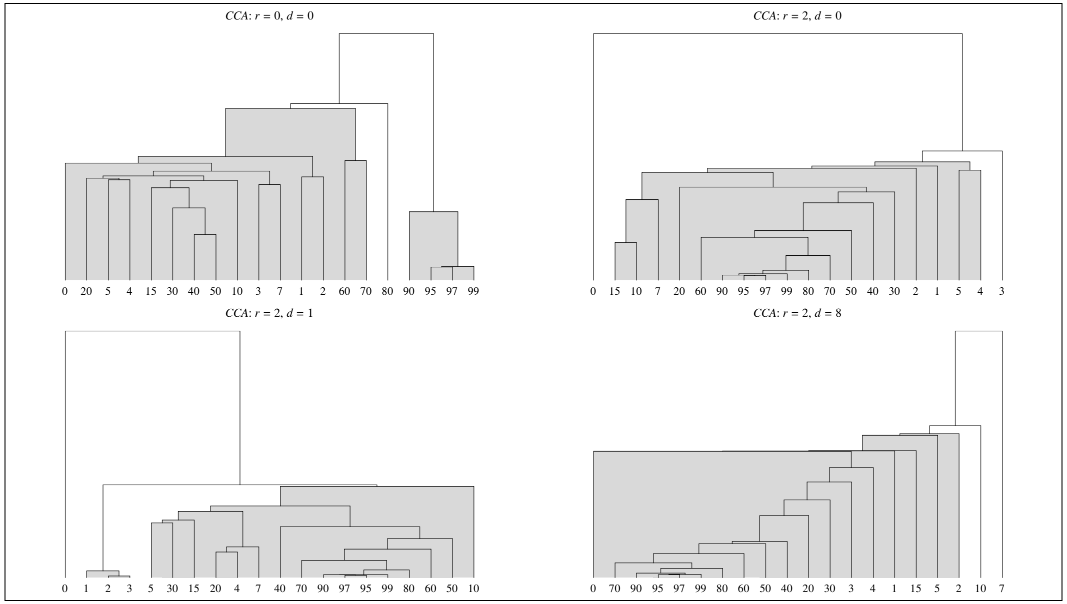
Total effect of MPA size (CCA)



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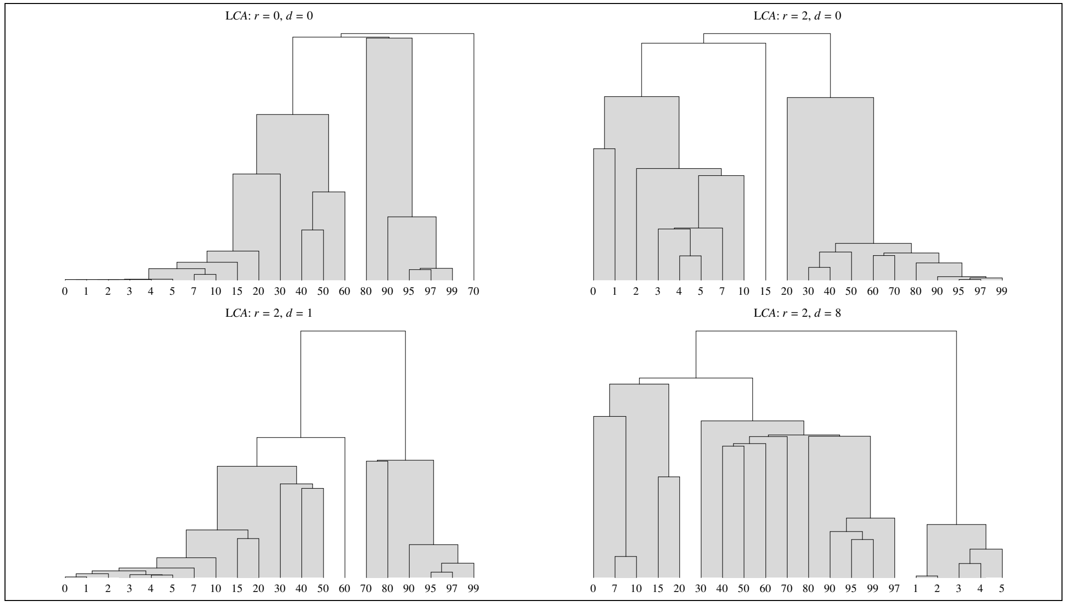
Total effect of MPA size (LCA)



Conlusion

- **Average biomass increases and variance decreases with increasing MPA size, indicating larger stock fluctuations at smaller MPAs (limit cycle patterns in LCA and pseudo random patterns in CCA)**
- **Reduced fluctuation reduces the overall net revenue**
- **While effectively targeting areas with higher fish densities, fishing effort is only slightly reduced as MPA increases in size**

Final remarks

- **Cellular automata modelling approach provides a powerful method of including spatial distribution and dynamics**
- **The shown model is easily modified to three dimensions**
- **The spatial distribution of fishing effort seems to be crucial for the stock development**

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